

Problem B 3. 2008 Ch. E. 453 Supplemental Project Problem

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10/28/2008

Title : Engineers Retirement Plan Options

Given :

\$ 500,000	Future Sum Needed = S		
25	Years = Age Now		
65	Years = Retirement Age		
Plan 'A'	6.03%	Interest Rate = Nominal Annual, Compounded Quarterly	4
Plan 'B'	5.98%	Interest Rate = Nominal Annual, Compounded Daily	365
\$ 57,800	Annual wages		
15.30%	FICA with holdings		
5.10%	Interest Rate = Nominal Annual, Compounded Annually		
4.0%	Inflation interest Rate = Annual		

Approach :

Years working to contribute = Retirement Age - Age Now	40	Years = contributing
Plan 'A' quarters contributed = quarters * years =	160	n
Plan 'B' quarters contributed = days * years =	14,600	n
(a) Future Worth of Annuity	$R = S [i / \{ (1+i)^n - 1 \}]$	\$756.99 per Quarter Plan 'A'
effective annual interest rate	$i_{\text{effective}} = [(1+i/m)^m] - 1$	6.167729%
(b) Future Worth of Annuity		\$8.25 per Day Plan 'B'
effective annual interest rate	$i_{\text{effective}} = [(1+i/m)^m] - 1$	6.161900%
(c) Annual annuity deposits are 15.30% of wages	\$ 8,843.40	
Future Worth of Annuity	$S = R [\{ (1+i)^n - 1 \} / i]$	\$1,094,712.36
Perpetual interest income is 5.1% of the Future Worth of the Annuity		
(d) Future Worth at inflation rate compounded annually	$S = P [(1+i)^n]$	
Year after retirement = Years working to contribute + 1	41	Years = contributing

Computations :

(a) Annuity, 4 annual equal payments for 40 years	$S = R [\{ (1+i)^n - 1 \} / \{ i \}]$	
	$R = S [i / \{ (1+i)^n - 1 \}]$	per quarter
	$R = \$500,000 [0.0151 / \{ (1+0.0151)^{160} - 1 \}]$	\$ 756.99 per Quarter Plan 'A'
Total payments per year = 4 * R		\$3,027.96 ANSWER
(b) Annuity, 365 annual equal payments for 40 years		per day
	$R = \$500,000 [0.00016 / \{ (1+0.00016)^{14600} - 1 \}]$	\$ 8.25 per Day Plan 'B'
Total payments per year : S = \$100,000 [(1+0.093)^5]		\$3,011.25 ANSWER

Observations :

Plans are very, very close to same values

Plan 'A' has a larger effective interest rate by	0.00583%	ANSWER
Plan 'B' has a lower sum of deposits by	\$16.71	ANSWER

but the complexity of making daily deposits (including weekends when banks are closed), time and effort is probably worth more than the \$17's

therefore select **Plan 'A'** ANSWER

(c) Annual annuity deposits are 15.30% of wages		
Future Worth of Annuity	$S = R [\{ (1+i)^n - 1 \} / i]$	\$1,094,712.36 ANSWER
Perpetual interest income = i * S		\$55,830.33 ANSWER
(d) Future Worth of today's wages at 4% inflation rate		
Future Worth of today's wages	$S = P * [(1+i)^n]$	\$288,598.95 ANSWER
Needed annual income to maintain today's purchasing power		

Observations :

note actual deposits into the FICA account	\$ 353,736.00
interest earnings of the account	\$740,976.36
note the effect of quarterly compounding on the FICA sum	\$ 1,143,065.22
increased interest earnings of the account	\$48,352.86