

Problem 7-3

Goal: Compare future values for various compounding schedules

Given: Annual deposits \$10,000 **A**
 Effective annual interest 0.08 **i_{effective}**
 Length 10 **n, years**
 (1) Compounded annually 1 **m**
 (2) Compounded weekly 52 **m**
 (3) Compounded continuous **e**

The equation: $F = A [((1 + i)^n - 1) / i]$ Table. 7-3 annuity factor

Approach: Determine interest rate per period by converting effective to nominal, then to per period
 Use annuity interest factor to determine future sum value
 Discount using compound interest equation to determine present worth of future sum

Calculations:

(1) Compounded annually 0.08 **i_{nominal}** since annual compounding effective = nominal = per period
 Factor (F/A, i, N) 14.48656 = $[(1 + 0.08)^{10} - 1] / 0.08$
 Future value, F \$144,865.62 = $10,000 [((1 + 0.08)^{10} - 1) / 0.08]$

Answer

\$144,866 at end of 10 years

Discounting the future sum value \$67,100.81 = $144,866 / (1 + 0.08)^{10}$
 Present worth factor (P/A, i, N) \$67,100.81 = $10,000 [((1 + 0.08)^{10} - 1) / 0.08 (1 + 0.08)^{10}]$

Answer

\$67,101 Present worth of all deposits

Calculations:

(2) Compounded weekly 7.7018% **i_{nominal}** = $(\text{EXP}(\text{LN}(0.08+1)/52))-1$ *52 as fraction 0.07701802
 approach was to solve eqn 7-9 for **r** check from nominal to **i_{effective}** = 0.08000000
 0.1481% **i_{week}** = 0.077018 / 52 0.00148112
 weekly deposits \$192.31 **\$/week** = \$10,000 / 52
 number of periods 520 **weeks** = 10 years * 52 weeks/year

Factor (F/A, i, N) 782.46752 = $[(1 + 0.00148)^{520} - 1] / 0.00148$
 Future value, F \$150,474.52 = $192.31 [((1 + 0.00148)^{520} - 1) / 0.00148]$

Answer

\$150,475 at end of 10 years

Discounting the future sum value \$71,652.82 = $150,475 / (1 + 0.08)^{10}$
 Present worth factor (P/A, i, N) \$69,698.82 = $192.31 [((1 + 0.00148)^{520} - 1) / 0.00148 (1 + 0.00148)^{520}]$

Answer

\$69,699 Present worth of all deposits

Calculations:

assuming continuous cash flow deposits, Table 7-5 factors

(3) Compounded continuous 7.6961% **r_{continuous}** = $(\text{EXP}(\text{LN}(0.08+1)/52))-1$ *52 0.07696104
 approach was to solve eqn 7-15 for **r**

Factor (F/ \bar{A} , r, N) 15.05859 = $[(e^{0.07696 * 10} - 1) / 0.07696] = [(e^{rN} - 1) / r]$
 Future value, F \$150,585.93 = $10,000 [(e^{0.07696 * 10} - 1) / 0.07696]$

Answer

\$150,586 at end of 10 years

Discounting the future sum value \$69,750.42 = $150,586 / e^{0.07696 * 10}$ Factor (P/F, i, j)
 Present worth factor \$69,750.42 = $10,000 [(e^{0.07696 * 10} - 1) / 0.07696 e^{0.0769 * 10}]$
 Factor (P/ \bar{A} , r, N)

Answer

\$69,750 Present worth of all deposits

Problem statement specifically says "effective (annual) interest rate is 8 percent"; but:

Alternatively: Using the stated 8% as the Nominal Rate (the way Text solution book did it)

(2) Compounded weekly 8.0000% i_{nominal} 0.08000000
 0.1538% $i_{\text{week}} = 0.08 / 52$ 0.00153846
 weekly deposits \$192.31 **\$/week** = \$10,000 / 52
 number of periods 520 **weeks** = 10 years * 52 weeks/year

Factor (F/A, i, N) 795.71257 = $[(1 + 0.001538)^{520} - 1] / 0.001538$
 Future value, F \$153,021.65 = $192.31 [(1 + 0.001538)^{520} - 1] / 0.001538$

Answer

\$153,022 at end of 10 years

Discounting the future sum value \$70,878.63 = $150,475 / (1 + 0.08)^{10}$

Present worth factor \$68,799.34 = $192.31 [(1 + 0.00148)^{520} - 1] / 0.00148 (1 + 0.00148)^{520}$

Answer

\$68,799 Present worth of all deposits

**Alternatively: Using the stated 8% as the Nominal Rate (the way Text solution book did it)
 assuming continuous cash flow deposits, Table 7-5 factors**

(3) Compounded continuous 8.0000% $r_{\text{continuous}} = r_{\text{effective}}$ 0.08000000

Factor (F/Ā, r, N) 15.31926 = $[e^{0.08 * 10} - 1] / 0.08$ = $[(e^{rN} - 1) / r]$

Future value, F \$153,192.62 = $10,000 [(e^{0.08 * 10} - 1) / 0.08]$

Answer

\$153,193 at end of 10 years

Discounting the future sum value \$68,833.88 = $150,586 / e^{0.07696 * 10}$

Present worth factor \$68,833.88 = $10,000 [(e^{0.07696 * 10} - 1) / 0.07696 e^{0.0769 * 10}]$

Answer

\$68,834 Present worth of all deposits