OPTIMUM METHODS for SINGLE VARIABLE SEARCHES

FIBONACCI Search

Based on numerical sequence $X_n = X_{n-1} + X_{n-2}$

Fibonacci numbers: F_N

$F_0 = 1$	$F_5 = 8$	$F_{10} = 89$	$F_{15} = 987$	$F_{20} = 10,946$
$F_1 = 1$	$F_6 = 13$	$F_{11} = 144$	$F_{16} = 1,597$	$F_{21} = 17,711$
$F_2 = 2$	$F_7 = 21$	$F_{12} = 233$	$F_{17} = 2,584$	$F_{22} = 28,657$
$F_3 = 3$	$F_8 = 34$	$F_{13} = 377$	$F_{18} = 4,181$	$F_{23} = 46,368$
$F_4 = 5$	$F_9 = 55$	$F_{14} = 610$	$F_{19} = 6,765$	$F_{24} = 75,025$

Search Procedure:

1. Select number of experiments, ***N***
2. Select upper and lower region constraint, region length, ***L***, becomes:

$$L_1 = X_{UPPER} - X_{LOWER}$$

3. Evaluate function at two points equidistant from each end of the interval, this distance is determined from:

$$d_1 = [F_{N-2} / F_N] * L_1$$

Therefore evaluate function at:

$$(\text{Lower limit} + d_1) \quad \text{and} \quad (\text{Upper limit} - d_1)$$

4. Eliminate a region based on presumed unimodality, region now has new upper or lower limit

5. Repeat step 3 with d_2 and new region L_2

note, one point within region is known from previous step

General term:

$$d_i = \{ F_{[N - (i + 1)]} / F_{[N - (i - 1)]} \} * L_i$$

Fraction of original interval remaining = $1.0 / F_N$

Example: $1/F_{24}$ = *0.0000133 or 0.0013 %*

Disadvantage of Fibonacci Search is that ***N*** must be known at start.

But, as ***N*** gets large, F_{N-2} / F_N converges to **0.381966012**

↳ $\left[\frac{3 - \sqrt{5}}{2} \right]$ ↥ ***Golden Ratio***

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GOLDEN SECTION Search

Golden Section is based on convergence of Fibonacci Search ratio ≈ 0.382 ***Golden Ratio***

Therefore $d_1' = 0.381966 * L$

and $d_1'' = (1 - 0.381966) * L = 0.618034 * L$

Search Procedure:

1. Select upper and lower region constraints

   $$L = X_{UPPER} - X_{LOWER}$$

2. Evaluate function at two points

   $$X = X_{LOWER} + 0.382 * L \quad \text{and} \quad X = X_{LOWER} + 0.618 * L$$

3. Eliminate a region
4. Recalculate interval L
5. Repeat step 2, except one of the points is already known depending upon the region eliminated
6. Continue until desired accuracy is obtained

Fraction of original interval remaining $= (0.618)^{(N - 1)}$

Example: $N = 24$, *Fraction remaining* $= (0.618)^{(23)} = 0.0000156$ *or* $0.0016\,\%$

References:

Rudd and Watson, Strategy of Process Engineering,
John Wiley & Sons, 1968, Chapter 6.

Kuester and Mize, Optimization Techniques with FORTRAN,
McGraw-Hill, 1973, Chapter 8.